MOTION OF THE SURFACE SEPARATING TWO LIQUIDS OF FINITE DEPTH UNDER THE INFLUENCE OF VARIABLE FORCE FIELDS

V. S. Sizonov

We consider the motion of the surface separating two liquids in a nonstationary gravitational field or, equivalently, under the action of a variable acceleration (an overload) perpendicular to the surface. Conditions are derived for the instability of the surface separating two liquids of finite depth in periodic, constant, and impulsive force fields; the physical mechanism acting during the development of instability is studied, and some features of the onset of instability are investigated for the case in which the space is bounded.

The behavior of a nonviscous incompressible liquid under the action of a constant acceleration perpendicular to the free surface was considered by Taylor [1]. He showed that acceleration directed from the heavier to the lighter liquid has a stabilizing effect on disturbances of the free surface; while acceleration in the opposite direction tends to increase instability and leads to unlimited increases in small disturbances. This last effect is called Taylor instability.

1. Let two nonviscous incompressible liquids fill a horizontal layer bounded by two infinite parallel planes. Let the origin of a rectangular coordinate system be on the surface separating the liquids, with the axis in the direction of the interface and the y axis directed vertically upwards. The depth of the lower liquid is h_1 (the subscript 1 always indicates quantities for the lower liquid), the depth of the upper liquid h_2. Motion is assumed to be two-dimensional and a surface tension T acts at the liquid interface. Let disturbances of the interface surface $\eta(x, t)$ be of the form

$$\eta = -a \cos mx\, \sigma(t) \tag{1.1}$$

where the amplitude a and the wave number m are real and positive, and $\sigma(t)$ is a function of the time to be determined.

The boundary conditions for the problem are as follows:

$$-\frac{\partial \varphi_1}{\partial y}\Big|_{y=-h_1} = -\frac{\partial \varphi_2}{\partial y}\Big|_{y=h_2} = 0$$

on the solid walls, and

$$\frac{\partial \eta}{\partial t} + (v\nabla)\eta = 0, \qquad v = -\nabla\varphi$$

on the interface surface.

Here v is the velocity of the liquid and φ is the velocity potential. The last boundary condition can be simplified if it is assumed that η and $\nabla\eta$ are small, since then the convection term in the equation of the surface may be neglected and we have

$$-\frac{\partial \varphi_k}{\partial y}\Big|_{y=\eta} = \frac{\partial \eta}{\partial t} \qquad (k = 1, 2)$$

Rostov-on-Don. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 55-62, September-October, 1970. Original article submitted December 2, 1969.

We now introduce the complex potentials

$$w_k(z) = \varphi_k + i\psi_k \qquad (z = x + iy)$$

We seek these potentials in the form

$$w_1 = \cos m\,(z + ih_1)\, b_1(t), \quad w_2 = -\cos m\,(z - ih_2)\, b_2(t)$$

The linearized boundary conditions on the interface are

$$b_1(t) = \frac{a}{m\,\mathrm{sh}\, mh_1}\frac{d\sigma}{dt}, \qquad b_2(t) = \frac{a}{m\,\mathrm{sh}\, mh_2}\frac{d\sigma}{dt}$$

and so the velocity potentials are

$$\begin{aligned}\varphi_1 &= \frac{a}{m\,\mathrm{sh}\, mh_1}\cos mx\,\mathrm{ch}\, m\,(y + h_1)\frac{d\sigma}{dt}\\ \varphi_2 &= -\frac{a}{m\,\mathrm{sh}\, mh_2}\cos mx\,\mathrm{ch}\, m\,(y - h_2)\frac{d\sigma}{dt}\end{aligned} \tag{1.2}$$

It is easily verified that these expressions satisfy the Laplace equation $\nabla^2\varphi_k = 0$ and the boundary conditions.

To find the dynamic condition on the interface, we consider Euler's equation, written as

$$\frac{\partial v_k}{\partial t} + (v_k\nabla)\, v_k = j - \frac{1}{\rho}\nabla p_k$$

where j is the field of the external forces. Assuming that this field is nonstationary and conservative, i.e., that it has a potential, we write

$$j = -g\beta(t) = -\Delta\Pi\beta(t)$$

where g and Π ($g = \nabla\Pi$) are functions of the coordinates but not of the time. Representation of the field in this way is equivalent to the assumption that the ratio of the field strengths at any two points of space is independent of the time. The function $\beta(t)$ describes the variation of the field with time at any point of space; in general no restrictions are imposed on this function, and it describes fields that are suddenly applied, that gradually weaken, that are impulsive or time-periodic, etc. Since $v_k = -\nabla\varphi_k$, Euler's equations imply

$$\nabla\left(\frac{p_k}{\rho_k} + \frac{1}{2}\nabla\varphi_k\nabla\varphi_k + \Pi\beta(t) - \frac{\partial\varphi_k}{\partial t}\right) = 0$$

Hence the Cauchy–Lagrange integral is

$$p_k + \frac{1}{2}\rho_k\nabla\varphi_k\nabla\varphi_k + \rho_k\Pi\beta(t) - \rho_k\frac{\partial\varphi_k}{\partial t} = f(t)$$

where $f(t)$ is an arbitrary function of the time.

Since $\nabla\eta$ is small, the Laplace's condition for the surface tension T is

$$p_1 - p_2 = -T\frac{\partial^2\eta}{\partial x^2}$$

The last two equations supply the dynamic condition on the interface:

$$\frac{\partial}{\partial t}(\rho_1\varphi_1 - \rho_2\varphi_2) - \left[(\rho_1 - \rho_2)\, g\beta(t) + \frac{1}{2}(\rho_1\nabla\varphi_1\nabla\varphi_1 - \rho_2\nabla\varphi_2\nabla\varphi_2)\right]\eta + T\frac{\partial^2\eta}{\partial x^2} = 0 \tag{1.3}$$

Now substitute the expressions for φ_1, φ_2 and η in (1.1) and (1.2) in (1.3). Using the fact that η and $\eta\nabla$ are small and neglecting the term $\frac{1}{2}(\rho_1\nabla\varphi_1\nabla\varphi_1 - \rho_2\nabla\varphi_2\nabla\varphi_2)$, we obtain

$$\begin{gathered}\frac{d^2\sigma}{d\tau^2} + [(1 - \rho)\,\beta(\tau) + B^{-1}]\,\sigma = 0\\ \tau = \sqrt{mgh}t, \quad \rho = \frac{\rho_2}{\rho_1}, \quad h = (\mathrm{cth}\, mh_1 + \rho\,\mathrm{cth}\, mh_2)^{-1}, \quad B = \frac{g\rho_1}{m^2T}\end{gathered} \tag{1.4}$$

Here τ is the dimensionless time and B is the Bond number. Solutions of (1.4) for specific functional relations between the external field and the time $\beta(\tau)$ yield the required functions $\eta(\tau)$ describing the variation of the interface shape with time.

2. Equation (1.4) is a second-order linear differential equation with invariant $I(\tau) = (1-\rho)\,\beta(\tau) + B^{-1}$.

Comparison theorems can be used to establish several properties of solutions of this equation, i.e., to deduce the properties of the motion with time of the interface from properties of the invariant $I(\tau)$.

1. If $I(\tau) \le 0$ everywhere in the interval (τ_1, τ_2), then all solutions of the equation are nonoscillatory and have not more than one zero in this interval. This is the condition for the continuous entrainment of interface displacement during the time interval. If $I(\tau) \le 0$ for all τ and $I(\tau) \not\equiv 0$, all solutions except the trivial solution increase unboundedly. In this sense, the condition $I(\tau) \le 0$ is sufficient for the interface to be unstable.

2. If

$$I(\tau) > 0 \qquad (I(\tau) \neq \text{const})$$

in the closed interval $[\tau_1, \tau_2]$, the solutions of (1.4) are oscillatory. If

$$s = \inf I(\tau), \qquad S = \sup I(\tau)$$

in $[\tau_1, \tau_2]$, then the distance between two successive zeros is smaller than $\pi/\sqrt{s}$ but larger than $\pi/\sqrt{S}$.

3. If $I(\tau) = c > 0$ on an unbounded interval $\tau > \tau_1$, every solution has an infinite number of zeros (the interface crosses its equilibrium position an infinite number of times).

If $I(\tau)$ is continuously differentiable and monotonic, the amplitude of each solution increases monotonically when $I(\tau)$ decreases, and decreases for increasing $I(\tau)$.

4. If $I(\tau) \to d_1^2 > 0$ when $\tau \to \infty$, solutions of (1.4) behave for large τ like solutions of

$$\frac{d^2\sigma}{d\tau^2} + d_1^2\sigma = 0$$

5. If $I(\tau) \to 0$ when $\tau \to \infty$ and, starting with some value of τ, the condition

$$0 \leqslant I(\tau) \leqslant {}^1/_4\,\tau^{-2}$$

holds, then a solution of (1.4) cannot have an infinite number of zeros (starting at some instant, the interface no longer passes through its equilibrium position). For comparison we use Euler's equation

$$\frac{d^2\sigma}{d\tau^2} + \frac{k^2}{\tau^2}\sigma = 0$$

6. If $(1-\rho)\,\beta(\tau) = O(x^{-k})$, where $k > 1$, then every nontrivial solution $y(\tau) \not\equiv 0$ can be written

$$y = \Gamma(\tau)\sin[B^{-1}\tau + E(\tau)], \quad y' = B^{-1}\Gamma(\tau)\cos[B^{-1}\tau + E(\tau)]$$
$$(B > 0)$$

where $\Gamma(\tau)$ and $E(\tau)$ are differentiable functions and the prime indicates differentiation with respect to τ. With the appropriate choice of E_0 and $\Gamma_0 \neq 0$, these functions are

$$\Gamma(\tau) = \Gamma_0 + O(x^{-k+1}), \quad E(\tau) = E_0 + O(x^{-k+1})$$

7. If $\beta(\tau)$ is continuously differentiable for $\tau > \tau_1$ and $(1-\rho)\,\beta(\tau) = O(x^{-1})$ $\beta'(\tau) = O(x^{-2})$ for $x \to \infty$, then sup lim $|y_B(\tau)|$ for $x \to \infty$, where y_B is any solution of (1.4) corresponding to a given value of B, lies between limits independent of B for all B_* satisfying $B \ge B_* > 0$.

8. If $I(\tau)$ is continuous, negative, and periodic with period $\omega > 0$, then the general solution of (1.4) has the form

$$c_1\exp(\alpha_1\tau)\,\sigma_1(\tau) + c_2\exp(\alpha_2\tau)\,\sigma_2(\tau)$$
$$\alpha_1 = \frac{\ln\rho_1}{\omega} > 0, \quad \alpha_2 = \frac{\ln\rho_2}{\omega} < 0, \quad \rho_1 > 1, \quad 0 < \rho_2 < 1$$

Here C_1 and C_2 are arbitrary constants, ρ_1 and ρ_2 are the roots of the characteristic equation, and $\sigma_1(\tau)$, $\sigma_2(\tau)$ are functions with period ω.

If $I(\tau) > 0$ for all positive λ such that

$$0 < \frac{\omega\lambda}{2}\int_0^\omega p(u)\,du \leqslant 2$$

then the general solution is of the form

$$c_1\sigma_1(\tau) + c_2\sigma_2(\tau)$$
$$\sigma_1(\tau) = \cos\frac{\theta\tau}{\omega}\psi_1(\tau) - \sin\frac{\theta\tau}{\omega}\psi_2(\tau)$$
$$\sigma_2(\tau) = \cos\frac{\theta\tau}{\omega}\psi_2(\tau) + \sin\frac{\theta\tau}{\omega}\psi_1(\tau)$$

Here C_1 and C_2 are arbitrary constants and $\psi_1(\tau)$ and $\psi_2(\tau)$ are functions with period ω. Hence solutions are unstable in this case.

3. In the simplest case, in which $\beta(\tau)$=const(independent of the time), Eq. (1.4), depending on the value of B, can have three types of solution:

$$\sigma(\tau) = A_1\cos(\tau\sqrt{(1-\rho)\beta + B^{-1}}) + A_2\sin(\tau\sqrt{(1-\rho)\beta + B^{-1}})$$
$$\text{for}\quad B^{-1} > (\rho - 1)\beta$$
$$\sigma(\tau) = A_1 + A_2\tau \quad\text{for}\quad B^{-1} = (\rho - 1)\beta$$
$$\sigma(\tau) = A_1\,\mathrm{ch}(\tau\sqrt{|(1-\rho)\beta + B^{-1}|}) + A_2\,\mathrm{sh}(\tau\sqrt{|(1-\rho)\beta + B^{-1}|})$$
$$\text{for}\quad B^{-1} < (\rho - 1)\beta$$

The second type corresponds to the critical value $B_* = [\beta(\rho - 1)]^{-1}$, separating stable and unstable interface states in constant gravitational fields.

The critical value $B_* = [\beta(\rho - 1)]^{-1}$ corresponds to an unstable state.

Such motion is realized, for example, during the transition of a surface-tension free system (for example a system consisting of two gases) into a weightless state. If there are waves on the interface in the state with gravitation, then the rate of growth of disturbances depends on the phase of the disturbances at the onset of weightlessness.

If the interface is passing through an equilibrium state at this time, then the rate of growth of disturbances will be greatest if the deviation of the surface from the equilibrium takes its largest value (the particle velocity is zero), and at the onset of weightlessness the interface "freezes" and the deviations from the equilibrium position will not increase with time.

For a liquid with surface tension, property 4 of Sec. 2 shows that the onset of weightlessness corresponds theoretically to states of stable interface oscillation.

However, this conclusion is of limited applicability, since the transition to weightlessness from a state of strong gravitation can cause the amplitude of waves on the interface to increase so rapidly that the initial fluid mass breaks up into separate volumes not connected with one another. Thus the transition to weightlessness of a liquid with surface tension leads to stable oscillating regimes of the free surface with amplitudes of the order of the amplitude of gravitational waves, unless the phase of surface oscillations at the onset of weightlessness corresponds to a deviation of the free surface from its equilibrium position close to maximal. Disintegration of the liquid may occur in the absence of surface tension.

A similar result holds in the more general case of sufficiently strong decreases in the intensity of the gravitational field.

Under constant positive overloads, the interface oscillations are standing waves whose frequencies increase with increasing external field strength (for constant wave number m).

Finally, the case $B^{-1} < (\rho - 1)\beta$ corresponds to Taylor instability if the negative overload satisfies the condition

$$-j > \frac{m^2 T}{\rho_1 - \rho_2}, \qquad j < 0$$

Here interface disturbances increase unboundedly at a rate determined by the value of B. The condition

$$\frac{d}{dm}\sqrt{|(1-\rho)\beta + B^{-1}|} = 0$$

determines the equation for the wave number m corresponding to the most rapid rate of increase of disturbances:

$$(h_0 + m_0 H_0)^{-1}(3h_0 + m_0 H_0) = -B$$
$$H_0 = h_1 \operatorname{csch}^2 m_0 h_1 + \rho h_2 \operatorname{csch}^2 m_0 h_2$$

where h_0 is the value of h for $m = m_0$.

For $h_1 \to \infty$, $h_2 \to \infty$, the maximum instability occurs for

$$(\rho - 1)\beta B = -3, \quad \text{or} \quad m_0 = \sqrt{1/3(-j)(\rho_1 - \rho_2)/T}$$

If the upper fluid is a gas $\rho \to 0$. In this case, when the depth of the lower fluid decreases ($h_1 \to 0$) the value of B corresponding to maximal instability increases until $(1-\rho)\beta B = -2$, or $m_0 \to \sqrt{1/2(-j)\rho_1/T}$ i.e., the increase in m_0 is $\sqrt{3/2}$ times its increase for a deep fluid. Hence with decreasing fluid depth the wavelength of the "most dangerous" waves, corresponding to the fastest rate of growth of disturbances, decreases.

4. We note that, in all the cases considered, the interface is a vortex surface, although the motion of each of the fluids separately is irrotational. This can be demonstrated by using the properties of acyclic motion. In the case of standing waves, for example, the difference between the horizontal velocity components in the two fluids at the interface is

$$-\left.\frac{\partial\varphi_1}{\partial x}\right|_{y=\eta} + \left.\frac{\partial\varphi_2}{\partial x}\right|_{y=\eta} = a\omega \sin mx\,(\operatorname{cth} mh_1 + \operatorname{cth} mh_2)\cos\omega t$$

and the upper and lower fluids move along the interface in opposite directions.

The maximum velocity difference is at nodes of standing waves; at antinodes the flow is irrotational.

It follows that, for any wave motion of the interface and large wave numbers m, Helmholz instability can occur at nodes of standing waves corresponding to small m, this instability arising sooner on a shallow liquid than on a deep liquid. This conclusion is confirmed by experimental studies of the development of Taylor instability in gases [2].

5. Taylor instability has the following simple interpretation. Suppose that the interface is wave-shaped and that, at a certain time, both fluids are motionless and the gravitational force is directed downwards. Then the pressure is constant in any horizontal plane above the interface in the upper fluid. However, in a horizontal plane below the interface the pressure varies periodically along the x axis. If the lower fluid is the heavier, then the pressure below wave crests is greater than below troughs. This pressure distribution will generate a flow in the lower fluid from crests to troughs, leading to a decrease in the deviation of the interface from its equilibrium position. In the opposite situation the flows are in the opposite direction, and the lower fluid begins to penetrate the upper fluid, flowing from below troughs to positions below crests.

6. In the case of three-dimensional disturbances of an interface between fluids in a bounded space, the velocity potential must satisfy Laplace's equation $\nabla^2\varphi_k = 0$ with the same boundary conditions used above, and also the supplementary condition $(\nabla\varphi_k)\mathbf{n} = 0$ on the lateral walls, where **n** is the normal to the surface. This extra condition imposes limitations on the waves numbers m, which here can only take a discrete set of values corresponding to the eigenvalues of the problem. For example, in a circular cylinder of radius R and height $h_1 + h_2$, the position of the interface can be described by

$$\eta = -aJ_n(mr)\cos n\theta\,\sigma(t) \tag{6.1}$$

where $J_n(mr)$ is the n-th order Bessel function.

Here the velocity potentials are

$$\varphi_1 = \frac{a}{m \operatorname{sh} mh_1} J_n(mr)\cos n\theta \operatorname{ch} m(y + h_1)\frac{d\sigma}{dt}$$
$$\varphi_2 = -\frac{a}{m \operatorname{sh} mh_2} J_n(mr)\cos n\theta \operatorname{ch} m(y - h_2)\frac{d\sigma}{dt}$$

The values of m are the roots of the equation

$$\left.\frac{dJ_n(mr)}{dr}\right|_{r=R} = 0 \quad \left(\frac{dJ_0(mr)}{dr} = -mJ_1(mr) \text{ for } n = 0\right)$$

Hence $m = \alpha_{1n}/R$, where α_{1n} takes values from the sequence 3.832, 7.016, 10.173... .

For a circular cylinder the minimum wave number m_* occurs for n = 1, when $m_* = 1.84/R$.

Hence an interface in a vertical, circular cylindrical tank becomes unstable if

$$-j > \left(\frac{1.84}{R}\right)^2 \frac{T}{\rho_1 - \rho_2}$$

For axially symmetric disturbances (n = 0), the admissible critical overload is greater by a factor $(3.832/1.84)^2 \approx 4.35$, and so in weak gravitational fields ($B \approx [(\rho - 1)\beta]^{-1}$) the loss of stability of the fluid surface is asymmetric with respect to the tank walls: On one side of the nodal diameter ($\theta = \pm \frac{1}{2}\pi$) only the lower fluid moves (upwards), and on the other side only the upper fluid moves (downwards), i.e., the liquid tilts about the diameter $\theta = \pm \frac{1}{2}\pi$.

In moderately strong fields, the wave number m_0 corresponding to maximal instability is determined by Eq. 1 of Sec. 2, and it may correspond to axially symmetric disturbances (the lower fluid penetrates the upper fluid on the tank axis, forcing the upper fluid downwards along the tank wall), and also to complex oscillation modes generating multiple "tongues" of one liquid penetrating the other.

The foregoing analysis is valid for a boundary-wetting angle $\frac{1}{2}\pi$; otherwise the undisturbed interface between the liquids in the tank is in general not plane. This reservation, however, is only essential for capillaries and, in general, for small values of B, when the interface may differ greatly from a plane for boundary-wetting angles differing considerably from $\frac{1}{2}\pi$. In moderately strong fields ($B \gg 1$), the undisturbed surface is nonplanar only close to the wall, the disturbance is small, and the results of the above analysis are not greatly altered even when the boundary angle is close to zero or to π.

7. We consider briefly one form of interface instability in variable force fields, namely the instability (which can be called parametric resonance) occurring in an oscillating force field.

If the gravitational field oscillates with time, the function $\beta(\tau)$ in (1.4) is time-periodic. Equation (1.4) reduces to Hill's equation, and in the special case in which the overloading is a sinusoidal function of the time it reduces to a Mathieu equation [3, 4]. If the period of the overload is equal to or is a multiple of $\pi\{[(1-\rho)\beta + B^{-1}]mgh\}^{-1/2}$, then parametric resonance arises and the interface disturbances increase unboundedly (if there is no damping) for arbitrarily small overload-amplitude oscillations. With increasing overload-amplitude oscillations, the range of unstable interface states widens in frequency, leading to instability not only for the frequencies indicated, but also for slightly different frequencies.

In the case of a sinusoidal force field, the range of instability for any oscillation amplitude can be determined by using Strett's diagram, for piecewise-constant overload by means of Meisner's method, and for other periodic oscillations by solving Eq. (1.4) numerically.

For instability the gravitational field acts on the interface as follows: The field becomes stronger when the interface is moving from a displaced position towards its equilibrium position, and becomes weaker when the interface is moving from its equilibrium position towards its maximum displacement. In this situation the field performs work on the system and increases the energy of the oscillations. Parametric resonance of the free surface of a liquid was observed by Faraday.

8. We conclude by considering impulsive fields. Suppose that an impulsive field engendered by a shock wave acts on the interface between two gases. In this case

$$\beta(t) = \frac{U}{g}\delta(t - t_1)$$

where U is the mass velocity at the interface caused by the passage of a shock wave and $\delta(t - t_1)$ is Dirac's function. Integration of this equation yields

$$\sigma(t) = \sigma_0 + \left[\frac{d\sigma}{dt}\bigg|_{t=0} - mh(1-\rho)U\sigma(t_1)\right]t, \quad \sigma_0 = \sigma(t)\,|_{t=0}$$

or, if the interface is at rest when the shock wave arrives,

$$\sigma(t) = \sigma_0\left(1 - m\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}Ut\right) \tag{8.1}$$

In (8.1) we have assumed that both gases extend to infinity. Hence the interface is unstable both if the shock wave is incident from the side of the lighter gas or from the side of the heavier gas; when the motion is from the side of the heavier gas ($\rho_1 > \rho_2$), the interface first moves to increase the sign of the disturbances (convexities become concavities and conversely), and then the deviations increase linearly with time. The rate of development of instability is faster when the difference between the gas densities is larger, when the amplitude of the initial interface disturbance is larger, and when the shock wave is stronger, while the rate of development is slower when the wavelength $\lambda = 2\pi/m$ is longer. R. Richtmyer [5] has attempted to solve this problem numerically from the point of view of Taylor instability, and experimental work has been done by E. E. Meshkov [6].

Equation (8.1) can be written

$$\frac{\sigma(t)}{\sigma(0)} = 1 - 2\pi \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} \frac{x}{\lambda}$$

Hence, all the straight lines in Fig. 5 in E. E. Meshkov's work cited above must emanate from a point with ordinate 1.

E. E. Meshkov's work confirms our conclusions. Hence, the behavior of the interface between two gases on the incidence of a shock wave is not determined by Taylor instability, but by a specific instability, analogous to that considered above, caused by the onset of weightlessness.

LITERATURE CITED

1. G. I. Taylor, "The instability of liquid surfaces when accelerated in a direction perpendicular to their planes," Proc. Roy. Soc., Ser. A, 201, No. 1065 (1950).
2. G. Birkhoff and E. Zarantonello, Jets, Wakes, and Cavities, Academic Press (1957).
3. T. B. Benjamin and F. Ursell, "The stability of the plane free surface of a liquid in vertical periodic motion," Proc. Roy. Soc., Ser. A., 225, 505-515 (1954).
4. N. N. Moiseev and V. V. Rumyantsev, Dynamics of Bodies with Cavities Containing a Liquid [in Russian], Fizmatgiz, Moscow (1965).
5. R. D. Richtmyer, "Taylor instability in shock acceleration of compressible fluids," Communs. Pure and Appl. Math., 13, 297-319 (1960).
6. E. E. Meshkov, "Instability of the interface between two gases accelerated by a shock wave," Izv. AN SSSR, MZhG, No. 5 (1969).